Three dimensional Eddington–inspired Born–Infeld gravity: solutions

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Eddington-inspired Born–Infeld gravity in three spacetime dimensions is studied. We obtain analytical and numerical solutions for the scale factor in a three dimensional Friedman-Roberston-Walker line element with (i)pressureless dust and (ii) a perfect fluid with $p = \frac{\rho}{2}$, as matter sources. Further, we also find a family of static, circularly symmetric analytical line elements with isotropic pressures. For FRW cosmology, we note that if the new parameter κ which arises in the theory, is positive ($\kappa > 0$), the solutions are singular (except for the open universe, with a specific condition), whereas if $\kappa < 0$, they represent non-singular spacetimes. On the other hand, in the circularly symmetric static case, our solutions are non-singular for both $\kappa > 0$ and $\kappa < 0$.

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I. INTRODUCTION

Theories of gravity different from General Relativity (GR) have been actively pursued by many, for a variety of reasons. One such reason relates to the possibility of avoiding the singularity problem in GR involving the occurrence of a big bang in cosmology or black holes in astrophysics. In a classical metric theory of gravity one is aware that these singularities are inevitable, as proved through the Hawking–Penrose theorems [1], under very general and physical assumptions. However, it is quite possible that in an alternate theory one might obtain non–singular geometries (for example non-singular Friedman–Robertson–Walker type cosmologies) as solutions–a feature which does not exist in the FRW cosmology based on GR. It may be noted that the removal/resolution of a singularity is also expected to be a basic feature of a quantum theory of gravity.

In this article, we look into one such alternate theory. Historically, its origin goes back to Eddington who showed us how de Sitter gravity could be obtained using an Einstein–Hilbert like action where $\sqrt{-g}R$ (Ricci scalar) is replaced by $\sqrt{-det(R_{ij})}$ [2]. Eddington's formulation had the advantage of choosing the connection as the basic variable, instead of the metric –therefore, it is essentially an affine formulation. However, coupling of matter remained a problem in this formulation.

We are also aware, in electrodynamics, of Born–Infeld theory [3], which was introduced in order to get rid of the infinity in the field at the location of the charge/current. A gravity theory in the metric formulation inspired by Born-Infeld electrodynamics was suggested by Deser and Gibbons [4]. Later Vollick [5] worked on the various aspects of the Deser-Gibbons proposal in a Palatini formulation and also introduced a non-trivial way of coupling matter in such theories. More recently, Banados and Ferreira [6] have come up with a formulation wherein the matter coupling is different and simpler from that introduced in Vollick's work [5]. We will focus in the theory proposed in [6] and call it as Eddington-inspired Born–Infeld (EiBI) gravity, for obvious reasons. Note that the EiBI theory has the feature that it reduces to GR, in vacuum.

It may be noted that the theory we consider falls within the class of bimetric theories of gravity (also called bi-gravity). The current bimetric theories have their origin in the seminal work of Isham, Salam and Strathdee [7]. Numerous papers on varied aspects of such bimetric theories have appeared in the last few years. The central feature here is the existence of a physical metric which couples to matter and another auxiliary metric which is not used for matter couplings. One needs to solve for both metrics through the field equations.

Let us now briefly recall Eddington–inspired Born–Infeld gravity. Since we deal with three spacetime dimensions in this article, we prefer to write down the action and ensuing field equations in three dimensional spacetime. The action for the theory developed in [6], is given as:

$$S_{BI}(g,\Gamma,\Psi) = \frac{2}{\kappa}\int d^3x \left[\sqrt{|g_{ij}+\kappa R_{ij}|} - \lambda\sqrt{-g}\right] + S_M(g,\Psi) \tag{1}$$

where $\Lambda = \frac{\lambda-1}{\kappa}$. Variation w.r.t Γ, done using the auxiliary metric $q_{ij} = g_{ij} + \kappa R_{ij}(q)$ gives

$$q_{ij} = g_{ij} + \kappa R_{ij}(q) \tag{2}$$

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Variation w.r.t g_{ij} gives

$$\sqrt{-q}q^{ij} = \lambda\sqrt{-g}g^{ij} - \kappa\sqrt{-g}T^{ij} \tag{3}$$

In order to obtain solutions, we need to assume a g_{ij} and a q_{ij} with unknown functions, as well as a matter stress energy (T^{ij}). Thereafter, we write down the field equations and obtain solutions using some additional assumptions about the metric functions and the stress energy.

Quite some work on various fronts have been carried out on various aspects of this theory in the last couple of years. Astrophysical aspects have been discussed in the references in [8] while cosmology in those cited in [9]. Other topics such as a domain wall brane has been analysed in [10]. More recently, generic features of paradigms on matter-gravity couplings have been discussed in [11]. However, in [12] a major problem related to surface singularities has been noticed which has put the theory on shaky ground insofar as stellar physics is concerned.

Our work here is reasonably modest. In the two subsequent sections, we discuss cosmological and circularly symmetric solutions in three spacetime dimensions, successively. In the final section, we briefly summarize and conclude. Most of our solutions are analytical and simple. They also maintain some of the generic features noted in the original work of Banados and Ferreira [6].

II. COSMOLOGY

Let us assume a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) line element in 2+1 dimensions, given as:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right] \tag{4}$$

where $k = +1, 0, -1$ for closed, flat and open universe respectively. The energy–momentum tensor is taken to be that of a fluid with $T^{ij} = (P+\rho)u^i u^j + P g^{ij}$. The conservation of energy–momentum leads to

$$\dot{\rho} = -2\frac{\dot{a}}{a}(P+\rho) \tag{5}$$

which implies $\rho \propto \frac{1}{a^2}$ (for $P = 0$) and $\rho \propto \frac{1}{a^3}$ (for $P = \frac{\rho}{2}$). Further, we assume the auxiliary line element to be of the form

$$ds_q^2 = -U(t)dt^2 + a^2(t)V(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right] \tag{6}$$

Using the auxiliary metric (q^{ij}), physical metric (g^{ij}) and stress-energy tensor (T^{ij}) in the field equation obtained by varying w.r.t. g_{ij}, we get two equations, given by,

$$U = \frac{D}{1+\kappa\rho_T} \tag{7a}$$

$$V = \frac{D}{1-\kappa P_T} \tag{7b}$$

where, $D = (1+\kappa\rho_T)(1-\kappa P_T)^2$, $\rho_T = \rho + \Lambda$, and $P_T = P - \Lambda$. We define two quantities $G_2(\rho,\Lambda)$ and $F_2(\rho,\Lambda)$, given as,

$$G_2(\rho,\Lambda) = \frac{1}{\kappa}\left(1 + U - \frac{2U}{V}\right) - \frac{2k}{a^2}\frac{U}{V} \tag{8a}$$

$$F_2(\rho,\Lambda) = 1 - \frac{\kappa(P_T+\rho_T)(1-w-\kappa P_T-\kappa w\rho_T)}{(1+\kappa\rho_T)(1-\kappa P_T)} \tag{8b}$$

where, in Eq. (8b), we have also assumed $P = w\rho$. Combining Eq. (2) and the results from Eqs. (8), we obtain the *Friedmann equation* given by,

$$H^2 = \frac{G_2}{2F_2^2} \tag{9}$$

Let us now examine two special cases: (i) *pressureless dust* $(w = 0)$ filled *flat* universe $(k = 0)$ with the cosmological constant, $\Lambda = 0$ and (ii) *radiation dominated* $(w = \frac{1}{2})$ flat universe with $\Lambda = 0$.
For the case (i), using the equation of state (Eq. (5)) and the Friedmann equation (Eq. (9)), we find

$$\left(\frac{\dot{\bar{\rho}}}{\bar{\rho}}\right)^2 = \frac{4\bar{\rho}(1+\bar{\rho})}{\kappa} \tag{10}$$

where, $\bar{\rho} = \kappa\rho$. Note that from the conservation law ρa^2 is a constant. Using this in Eq. (10), we obtain the solution for the scale factor $a(t)$ for $\kappa > 0$ as,

$$a(t) = \sqrt{C_1\left(\frac{t^2}{\kappa} - 1\right)} \tag{11}$$

where the constant $C_1 = |\bar{\rho}|\, a^2 > 0$. The solution clearly demonstrates that for $\kappa > 0$ the EiBI theory cannot avoid the initial singularity ($a(t)$ can become zero at a finite t and hence we have infinite curvature). However, for $\kappa < 0$, the Eq. (10) becomes,

$$\left|\frac{\dot{\bar{\rho}}}{\bar{\rho}}\right|^2 = \frac{4|\rho|(1-|\rho|)}{|\kappa|} \tag{12}$$

From Eq. (12), we note that there exists a maximum density (ρ_B) or, equivalently, a minimum value for the scale factor (a_B). The solution for the scale factor $a(t)$ for $\kappa < 0$ is given by,

$$a(t) = \sqrt{C_2\left(\frac{t^2}{|\kappa|} + 1\right)} \tag{13}$$

where C_2 is a constant. It is easy to see that the scale factor is never zero and thus, there is no curvature singularity. Both these solutions are plotted in the top row of Fig. 1.
For case (ii),(i.e radiation dominated universe), the Friedmann equation becomes:

$$H^2 = \frac{2}{\kappa}\left[2\bar{\rho} - 1 + \left(1 - \frac{\bar{\rho}}{2}\right)^2(1+\bar{\rho}) - \frac{\kappa k}{a^2}(2-\bar{\rho})\right]\frac{(1+\bar{\rho})(2-\bar{\rho})^2}{(4-\bar{\rho}+4\bar{\rho}^2)^2} \tag{14}$$

For the *flat universe* $(k = 0)$, the last term in the square-bracket of the right hand side of the Eq. (14) does not contribute and the equation becomes,

$$H^2 = \frac{2}{k}\left[2\bar{\rho} - 1 + \left(1 - \frac{\bar{\rho}}{2}\right)^2(1+\bar{\rho})\right]\frac{(1+\bar{\rho})(2-\bar{\rho})^2}{(4-\bar{\rho}+4\bar{\rho}^2)^2} \tag{15}$$

In this case, for $\kappa > 0$, $H^2 \geq 0$ for an arbitrary, but physically justifiable value of $\bar{\rho}$ (i.e for $\bar{\rho} \geq 0$). Thus, here also, a curvature singularity appears in the solution. Using the equation of state :$\bar{\rho}a^3 = C_3$, where C_3 is a constant, we can solve numerically the Eq. (15) for the time evolution of the scale-factor $a(t)$, which is shown in Fig. 1. However, for $\kappa < 0$, the Eq. (15) is rewritten as,

$$H^2 = \frac{2}{|\kappa|}\left[1 + 2|\bar{\rho}| + \left(1 + \frac{|\bar{\rho}|}{2}\right)^2(|\bar{\rho}| - 1)\right]\frac{(1-|\bar{\rho}|)(2+|\bar{\rho}|)^2}{(4+|\bar{\rho}|+4|\bar{\rho}|^2)^2} \tag{16}$$

The presence of the factor $(1 - |\bar{\rho}|)$ in the Eq. (16) leads to H^2 being negative when $|\bar{\rho}| > 1$. Therefore, in this case, for $\kappa < 0$, there exists a maximum density or, equivalently, a minimum, non–zero value for the scale factor.
Let us now define two dimensionless variables $\Omega = \frac{\rho}{\rho_B}$ $(0 \leq \Omega \leq 1)$,$z = \frac{a}{a_B}$. Using these we recast the equations as:

$$|\kappa H^2(\Omega)| = 2\left[1 + 2\Omega + \left(1 + \frac{\Omega}{2}\right)^2(\Omega - 1)\right]\frac{(1-\Omega)(2+\Omega)^2}{(4+\Omega+4\Omega^2)^2} \tag{17a}$$

$$\dot{z}^2 = \frac{2z^2}{|\kappa|}\left[1 + \frac{2}{z^3} + \left(1 + \frac{1}{2z^3}\right)^2\left(\frac{1}{z^3} - 1\right)\right]\frac{\left(1 - \frac{1}{z^3}\right)\left(2 + \frac{1}{z^3}\right)^2}{\left(4 + \frac{1}{z^3} + \frac{4}{z^6}\right)^2} \tag{17b}$$

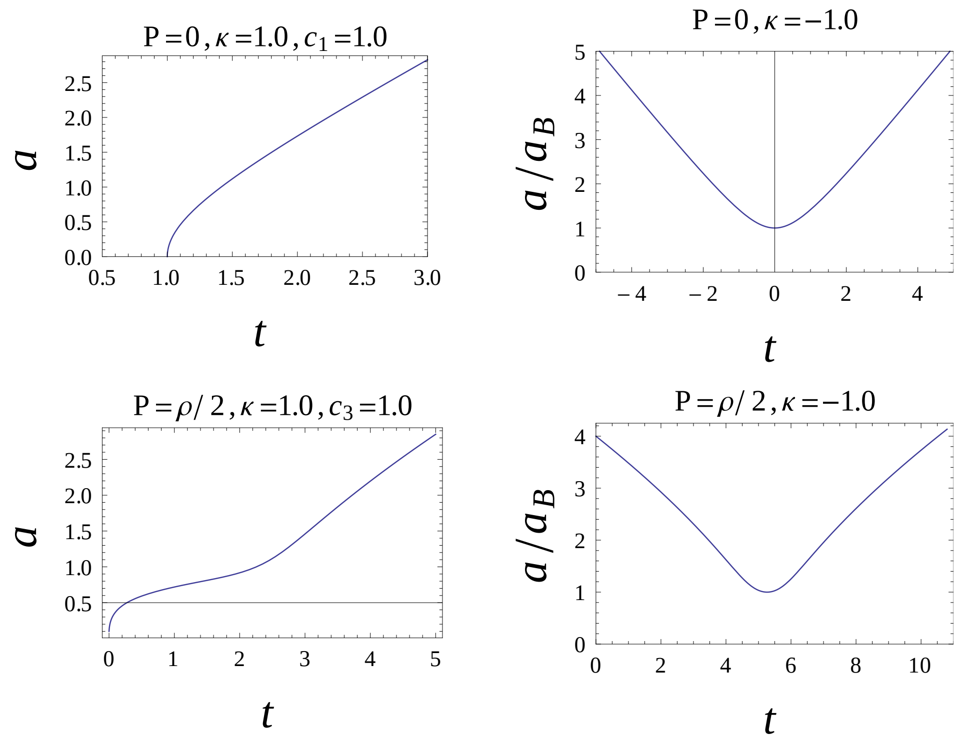

FIG. 1: Plot of the time evolution of the scale-factor $a(t)$ for flat universe $(k = 0)$.The parameters are specified on each frame. a_B is the minimum scale factor for non-singular universe. C_1 and C_3 are constants as defined in the text.

Numerically solving the above equation, we plot and note the time evolution of the scale factor using the Eqs. (17). In the bottom row of Fig. 1 we note a non-singular scale factor for $\kappa < 0$ –a result showing the existence of a bounce, similar to that obtained in $3 + 1$ dimensions. For $\kappa > 0$, the solution appears to be singular. The above solutions (especially, the ones for the $P = 0$ case) are instructive because they are, as far as we know, the only known analytical solutions in EiBI cosmology.

Introducing curvature in the spatial slices (i.e. $k = +1, -1$) does not yield anything drastically new in the dust $(P = 0)$ case primarily because of the fact that $\rho \propto \frac{1}{a^2}$.The solutions for $\kappa > 0$ and $\kappa < 0$ are given by,

$$a(t) = \sqrt{\left(1 - \frac{k}{\rho_0}\right) c_1 \left(\frac{t^2}{\kappa} - 1\right)}, \kappa > 0 \tag{18a}$$

$$a(t) = \sqrt{\left(1 - \frac{k}{\rho_0}\right) c_2 \left(\frac{t^2}{|\kappa|} + 1\right)}, \kappa < 0 \tag{18b}$$

where ρ_0 is a constant ($\rho a^2 = \rho_0$ for $P = 0$). For a $k = +1$ solution there is a lower bound on ρ_0 ($\rho_0 > 1$) whereas, for $k = -1$, ρ_0 is an arbitrary positive, real constant. When $k = 0$, we recover the earlier results (Eq. 11 and Eq. 13). It is easy to see that there is nothing new in the $P = 0, k \neq 0$ solutions.

However, in the radiation dominated ($P = \frac{\rho}{2}$) case we do get some interesting results, though the main conclusion regarding the appearance of a singularity or otherwise, is almost the same. The introduction of spatial curvature results in an additional term as shown in the square-bracket of the R.H.S. of Eq. (14) and this leads to all the differences. For a *closed universe* ($k = +1$), instead of a *bounce* we get an *oscillation* of the universe for $\kappa < 0$. For $\kappa > 0$ there is an additional feature implying a maximum value of the scale-factor, along with the singularity. These are shown in Fig.2. In an *open universe* ($k = -1$), we do not see any characteristic novelties for $\kappa < 0$, but for $\kappa > 0$ along with the singularity, we also get, under certain circumstances, a non-singular *loitering* phase of the early universe. This is shown through the plots in Fig. 3.

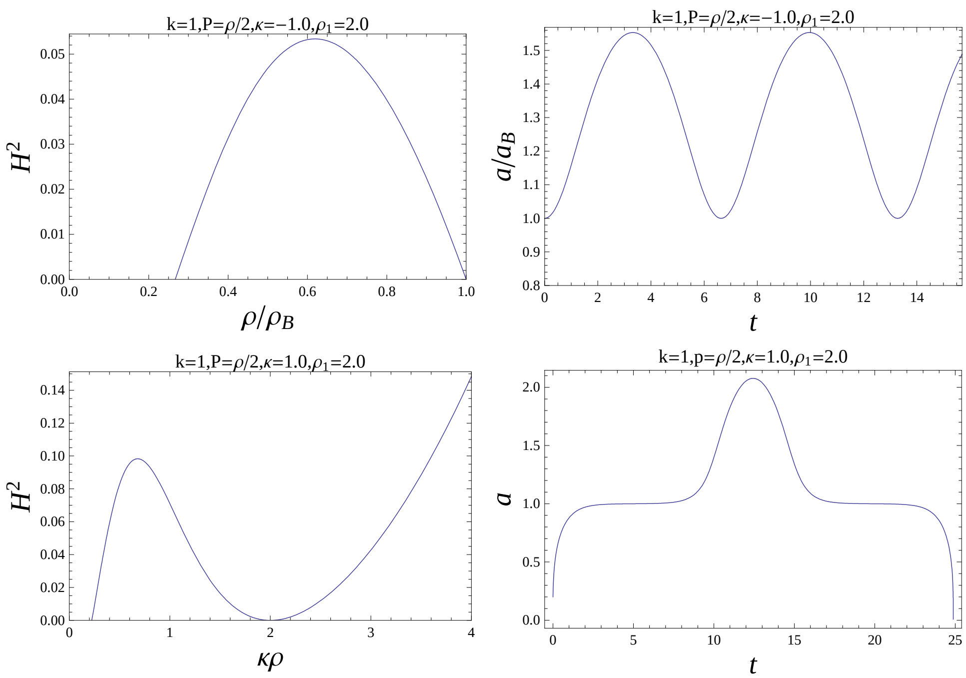

FIG. 2: Plot of H^2 and the time evolution of the scale-factor $a(t)$ for closed universe ($k = +1$). ρ_1 is a constant,($\rho a^3 = \rho_1$).

III. CIRCULARLY SYMMETRIC, STATIC SOLUTIONS

Let us now turn to a completely different class of line elements –i.e. those which are circularly symmetric and static. We consider a simple ansatz for the physical line element g_{ij},

$$ds^2 = -dt^2 + dl^2 + r^2(l)d\theta^2 \tag{19}$$

where $r(l)$ is a non–negative function and l extends from minus infinity to plus infinity. $r(l)$ represents the radius of a circle at each value of l. The energy-momentum tensor is assumed as, T^{ij}=diag.$\left(\rho, P_1, \frac{P_2}{r^2(l)}\right)$. Energy-momentum conservation, $\nabla_j T^{ij} = 0$, leads to the requirement, $P_1 = P_2 = P$. Let us further assume the auxiliary line element to be of the form

$$ds_q^2 = -dt^2 + u^2(l)dl^2 + v^2(l)r^2(l)d\theta^2 \tag{20}$$

where $u(l)$ and $v(l)$ are non–negative functions of l. The field equation obtained from g_{ij} variation yields,

$$P = \Lambda, \qquad u = v, \qquad \rho = \frac{u^2 - 1}{\kappa} - \Lambda \tag{21}$$

The other field equation obtained from Γ variation yields a single equation given as,

$$u^2 - 1 = -\kappa \left[\frac{r''}{r} + \frac{r'u'}{ru} + \frac{u''}{u} - \left(\frac{u'}{u}\right)^2\right] \tag{22}$$

Trivial solutions can be found by assuming $u = 1$ which gives $r(l) \sim l$ (flat spacetime) and $u = \beta$ ($\neq 1$ and a constant) which yields de Sitter spacetime in three dimensions.

Let us now rewrite the equation using the new variables U and S with $u^2 = U, r^2 = S$ in Eq. (22). The above equation then becomes

$$U - 1 = -\kappa \left[\frac{1}{2}\left(\frac{S'}{S} + \frac{U'}{U}\right)' + \frac{S'}{4S}\left(\frac{S'}{S} + \frac{U'}{U}\right)\right] \tag{23}$$

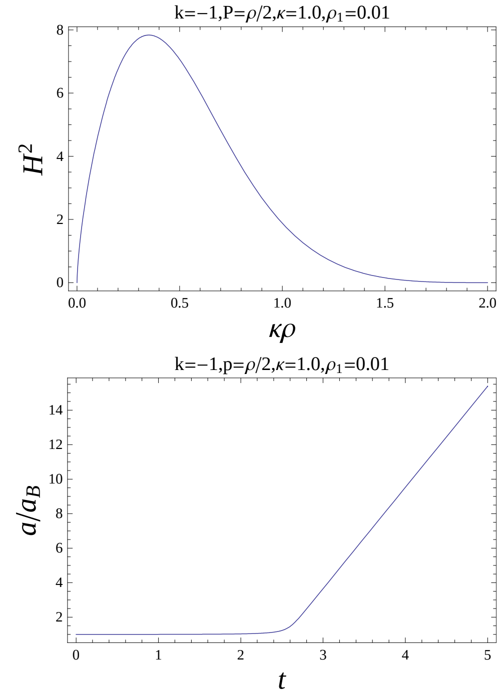

FIG. 3: Plot of H^2 and $a(t)$ for open universe ($k=-1$) showing the *loitering phase* at its early stage. ρ_1 is a constant,($\rho a^3 = \rho_1$).

To solve this equation we need to assume a relationship between U and S. Let us take

$$\frac{S'}{S} + \frac{U'}{U} = D \tag{24}$$

where D is a constant. Choosing D appropriately, we can transform the second order differential equation (Eq. 23) into an integrable ODE and solve it. Though different choices of D may give different mathematical solutions, only those are physically important which result in physically acceptable energy-density distributions (ρ_T). One such class of solutions is obtained by choosing D as a step function given by,

$$\begin{aligned} D &= -\frac{2\mu}{\sqrt{\kappa}}\left[\theta(l) - \theta(-l)\right] \qquad \text{for } \kappa > 0 \\ &= \frac{2\nu}{\sqrt{|\kappa|}}\left[\theta(l) - \theta(-l)\right] \qquad \text{for } \kappa < 0 \end{aligned} \tag{25}$$

where $\mu > 0$, $\mu \neq 1$, and $\nu \neq 0, 1$.
For $\kappa > 0$, we get the solution as,

$$S = S_0 \frac{\left[\exp\left(-\frac{2\mu|l|}{\sqrt{\kappa}}\right) - \mu^2 \exp\left(-\frac{2|l|}{\mu\sqrt{\kappa}}\right)\right]}{1-\mu^2} \tag{26a}$$

$$U = \frac{1-\mu^2}{1-\mu^2 \exp\left(-\frac{2|l|(1-\mu^2)}{\mu\sqrt{\kappa}}\right)} \tag{26b}$$

where, we have assumed $S(0) = S_0$ and $U(0) = 1$. The Ricci scalar (R for the physical metric) for such a solution is as follows:

$$R = -\frac{2\mu^2}{\kappa}\left[2\left[\frac{\exp\left(-\frac{2\mu|l|}{\sqrt{\kappa}}\right) - \frac{1}{\mu^2}\exp\left(-\frac{2|l|}{\mu\sqrt{\kappa}}\right)}{\exp\left(-\frac{2\mu|l|}{\sqrt{\kappa}}\right) - \mu^2\exp\left(-\frac{2|l|}{\mu\sqrt{\kappa}}\right)}\right] - \left[\frac{\exp\left(-\frac{2\mu|l|}{\sqrt{\kappa}}\right) - \exp\left(-\frac{2|l|}{\mu\sqrt{\kappa}}\right)}{\exp\left(-\frac{2\mu|l|}{\sqrt{\kappa}}\right) - \mu^2\exp\left(-\frac{2|l|}{\mu\sqrt{\kappa}}\right)}\right]^2\right] \tag{27}$$

We note that such solutions with $\kappa > 0$ are non-singular.
When $\kappa < 0$, we obtain the solution as,

$$S = S_0 \frac{\left[\exp\left(\frac{2\nu|l|}{\sqrt{|\kappa|}}\right) + \nu^2 \exp\left(-\frac{2|l|}{\nu\sqrt{|\kappa|}}\right)\right]}{1+\nu^2} \tag{28a}$$

$$U = \frac{1+\nu^2}{1+\nu^2 \exp\left(-\frac{2|l|(1+\nu^2)}{\nu\sqrt{|\kappa|}}\right)} \tag{28b}$$

For this solution, the Ricci scalar turns out to be,

$$R = -\frac{2\nu^2}{|\kappa|}\left[2\left[\frac{\exp\left(\frac{2\nu|l|}{\sqrt{|\kappa|}}\right) + \frac{1}{\nu^2}\exp\left(-\frac{2|l|}{\nu\sqrt{|\kappa|}}\right)}{\exp\left(\frac{2\nu|l|}{\sqrt{|\kappa|}}\right) + \nu^2\exp\left(-\frac{2|l|}{\nu\sqrt{|\kappa|}}\right)}\right] - \left[\frac{\exp\left(\frac{2\nu|l|}{\sqrt{|\kappa|}}\right) - \exp\left(-\frac{2|l|}{\nu\sqrt{|\kappa|}}\right)}{\exp\left(\frac{2\nu|l|}{\sqrt{|\kappa|}}\right) + \nu^2\exp\left(-\frac{2|l|}{\nu\sqrt{|\kappa|}}\right)}\right]^2\right] \tag{29}$$

For $\nu = -1$, Eq. (28a) becomes,

$$S = S_0 \cosh\left(\frac{2l}{\sqrt{|\kappa|}}\right) \tag{30}$$

The corresponding Ricci scalar,from Eq. 29 turns out to be

$$R = -\frac{2 + 2sech^2\left(\frac{2l}{\sqrt{|\kappa|}}\right)}{|\kappa|} \tag{31}$$

which is non-singular. The *Kretschmann scalar* is

$$K = R^{ijkl}R_{ijkl} = \frac{4}{|\kappa|^2}\left[2 - \tanh\left(\frac{2l}{\sqrt{|\kappa|}}\right)\right]^2 \tag{32}$$

and it is non-singular as well. Therefore, assuming $S_0 = r_0^2$, the physical line element, for this case ($\kappa < 0$ and $\nu = -1$), can be written as,

$$ds^2 = -dt^2 + dl^2 + r_0^2 \cosh\left(\frac{2l}{\sqrt{|\kappa|}}\right) d\theta^2 \tag{33}$$

One can also rewrite it as,

$$ds^2 = -dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\theta^2 \tag{34}$$

where,

$$b(r) = r + \frac{1}{|\kappa|}\left(\frac{r_0^4}{r} - r^3\right) \tag{35}$$

and $r = r_0\sqrt{\cosh\left(\frac{2l}{\sqrt{|\kappa|}}\right)}$. Thus, $r \geq r_0$ and a metric singularity occurs in the transformed metric function only at $r = r_0$. We plot the transformed metric function $1/(1 - \frac{b(r)}{r})$ and have shown it in Fig. 4. Note that this solution is not asymptotically flat (i.e. $\frac{b(r)}{r}$ does not tend to zero as $r \to \infty$). However, there is a minimum radius $r = r_0$ and the geometry is symmetric as $l \to -l$. Also $b(r = r_0) = r_0$. Thus, the features are similar to that of a Lorentzian wormhole [13] though the geometry is not asymptotically flat.

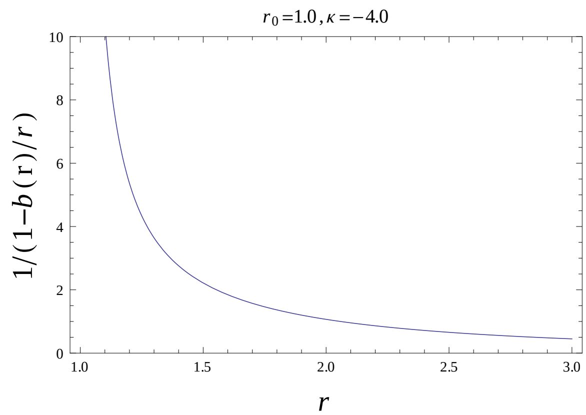

FIG. 4: Plot of $\frac{1}{1-\frac{b(r)}{r}}$

In Fig. 5 we have shown and compared the g and q metric functions, the Ricci scalar (R), and the energy-density (ρ_T) for two different solutions, one for $\kappa > 0$, $\mu = 2.0$ and the other for $\kappa < 0$, $\nu = -1.0$. Note that all the plots are symmetric as $l \to -l$. In Fig. 6 we have plotted the *Ricci scalar* of the g metric and the energy density for different solutions.

In Fig. 6(a), we have shown the solutions for $\kappa > 0$ with two different values of the parameter μ, one for $\mu = 0.5$ and the other for $\mu = 3.0$. In both solutions, we note that the Ricci scalar is symmetric and tends to a negative constant value for large positive or negative l–it is asymptotically AdS, but near the origin it is positive. Interestingly, for $\mu \to 0$, or $\mu \gg 1$, it tends to zero (asymptotically flat spacetime). The energy density is also symmetric as $l \to -l$ for both $\mu = 0.5, 3.0$. In fact, by suitably adjusting Λ one can maintain a positive energy density everywhere for all l.

In Fig. 6(b), we have shown two different $\kappa < 0$ solutions, one for $\nu = -2.0$ and the other one for $\nu = 2.0$. Here, we see that, for both solutions, the Ricci scalar is symmetric, negative everywhere and asymptotically AdS. Additionally, it can also be shown that, it is asymptotically flat for large negative value of ν ($\nu \ll -1$) or very small positive value of ν ($0 < \nu \ll 1$). In these solutions we also note the energy density to be symmetric as $l \to -l$. For $\nu = 2.0$, though ρ_T is positive for all l, it has a minimum value at $l = 0$ and it increases for $|l| > 0$ tending to a constant value for large $|l|$.

IV. CONCLUSIONS

The main purpose behind this paper was to find simple solutions of EiBI gravity in three dimensions. To this end we have found an analytical FRW cosmology for pressureless dust and analytical circularly symmetric solutions. We have also obtained the numerical solution for radiation, in the case of cosmology. Apart from a flat $k = 0$ spatial section, we have also explored the $k = +1$ and $k = -1$ FRW models. The cosmological solutions presented here seem to point to a generic feature that singularities are not present if $\kappa < 0$ whereas singularities do arise if $\kappa > 0$. It is not clear whether one can make any general statement on this based on the solutions we have found. In the class of circularly symmetric, static solutions we note that they are nonsingular for both $\kappa > 0$ and $\kappa < 0$.

One may also look for more general solutions in the circularly symmetric case, with a non-constant g_{00}. This is likely to lead to three dimensional black holes in EiBI theory. Finally, it will surely be useful to see if our solutions in this toy three dimensional theory can serve as viable pointers towards finding new analytical or numerically obtained geometries in the real four dimensional scenario.

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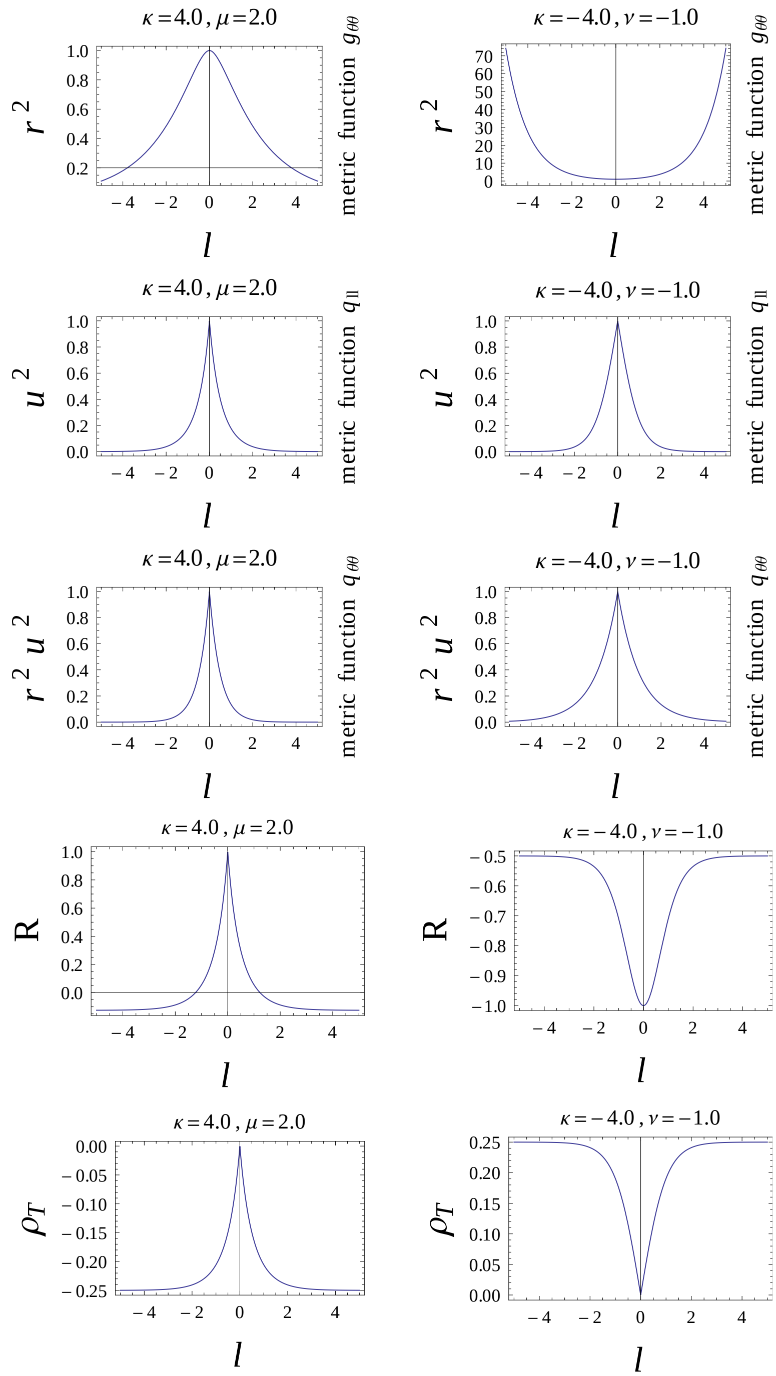

FIG. 5: Plot of the metric functions (in the physical and auxiliary line elements) , Ricci scalar(R) and energy density($\rho_T = \rho + \Lambda$) , for two different solutions with $\mu = 2.0$, $\kappa = 4.0$ and $\nu = -1.0$, $\kappa = -4.0$, assuming $r_0 = 1.0$

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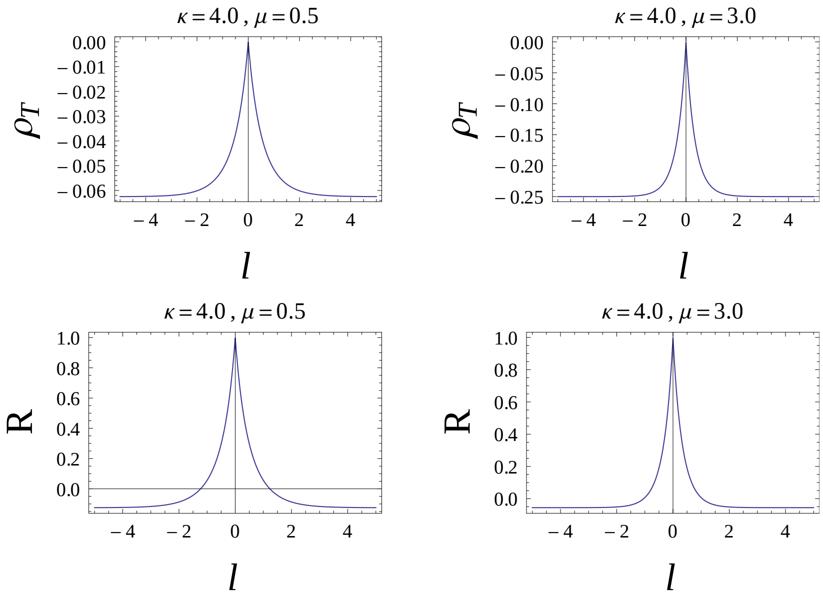
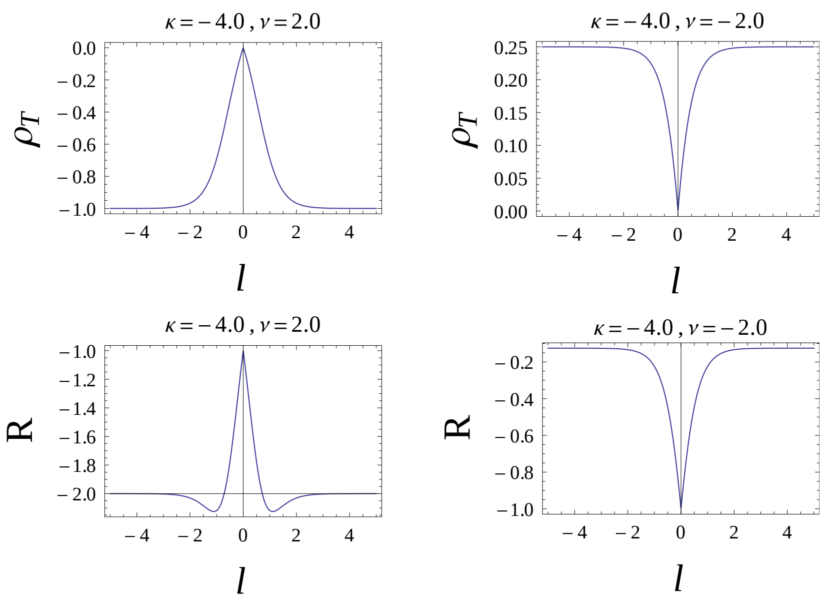


(a)For $\kappa > 0$, $\mu < 1.0$ (left) and $\mu > 1.0$ (right)



(b)For $\kappa < 0$, $\nu > 0.0$ (left) and $\nu < 0.0$ (right)

FIG. 6: Comparison of solutions with different energy-density and curvature scalar, for various values of the parameters (μ and ν).

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